A Hybrid Machine Learning Approach for Carbon Price Forecasting

New Techniques and Technologies for Statistics

Eurostat

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Introduction

Carbon Markets Essentials

What are Carbon Markets?

- Systems that enable the **trading of carbon emission allowances** to reduce greenhouse gas (GHG) emissions.
- They operate under **cap-and-trade systems**, where a limit is set on emissions, and companies can trade allowances.

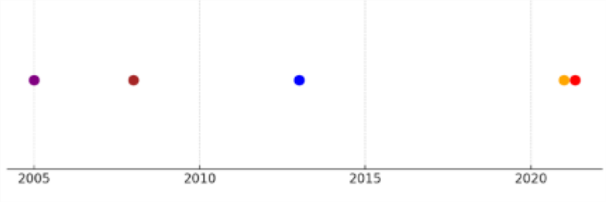
European Union Emissions Trading System (EU ETS)

- Established in **2005**, the **EU ETS** is the **largest carbon market globally**.
- Operates in **phases** to regulate emissions across industries.

United Kingdom Emissions Trading System (UK ETS)

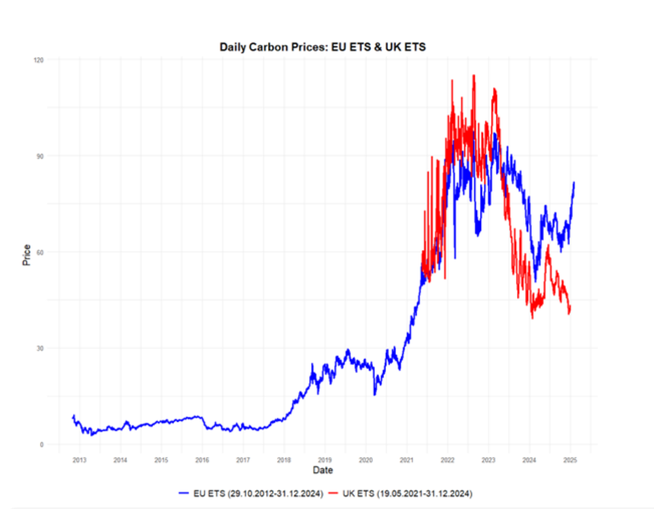
- Introduced in **May 2021**, post-Brexit, as a **replacement for the UK's participation in EU ETS**.
- Follows a **similar cap-and-trade structure** but with **UK-specific regulations**.
- Initially aligned with EU ETS but evolving **independently**.

Timeline of EU and UK ETS



- EU ETS Phase 1 Begins (2005)
- EU ETS Phase 2 Begins (2008)
- EU ETS Phase 3 Begins (2013)
- EU ETS Phase 4 Starts (2021)
- UK ETS Introduced Post-Brexit (2021)

Historical Carbon Prices for EU and UK ETS



Data and Descriptive Statistics

Data Overview

- Data Sources:
  - European Union Allowance (EUA) Daily Futures Contracts
  - United Kingdom Allowance (UKA) Daily Futures Contracts
  - Retrieved from **Refinitiv Eikon (ICE)**
- Time Series Coverage (to be extended):
  - EUA: January 2013 – August 2022 (2,479 observations)
  - UKA: May 2021 – August 2022 (346 observations)
  - Both time series aligned from 2013 to 2022 for consistency
- Key Characteristics:
  - EUA spans Phase 3 (2013-2020) & Phase 4 (2021-2030) of EU ETS.
  - **Post-Brexit UK carbon prices remain highly correlated with EU prices**.
  - **Log returns exhibit non-normality, fat tails, and left skewness**.

Descriptive Statistics

	mean	variance	skewness	kurtosis
Before splitting	0.001075	0.001090	-1.093903	20.952076
EU - After splitting	0.001764	0.001075	-0.876986	8.752500
UK - After splitting	0.001783	0.000737	-1.010363	7.885077

Figure 1: Empirical moments of log returns for EU and UK carbon prices: A pre- and post-separation perspective.

Descriptive Statistics

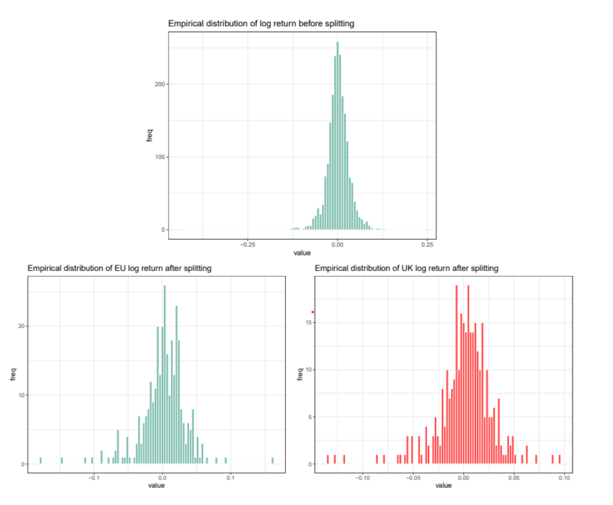


Figure 2: Plot of empirical moments of log returns for EU and UK carbon prices: A pre and post separation perspective.

Methodology

ARMA

ARMA (Autoregressive Moving Average)

- Statistical model for time-series forecasting.
- Captures **linear relationships** based on past values and moving averages.
- Best suited for **short-term forecasting** with stationary data.

ARMA

A general form of ARMA of orders (p, q) is specified as follows:

$$Y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i Y_{t-i} + \sum_{j=1}^{q} \beta_j \epsilon_{t-j} + \epsilon_t$$

where:

- Y_t is the dependent variable and Y_{t-i} its lagged values.
- ϵ_t is random noise, usually assumed to be normally distributed with zero mean.
- ϵ_{t-j} represents lagged residuals.

LSTM

- **Long Short-Term Memory (LSTM)** is a type of **Recurrent Neural Network (RNN)** designed to capture **long-term dependencies** and **non-linear patterns** in sequential data.
- Unlike ARMA, LSTM can adapt to **complex relationships** and **changing patterns** in time-series data.

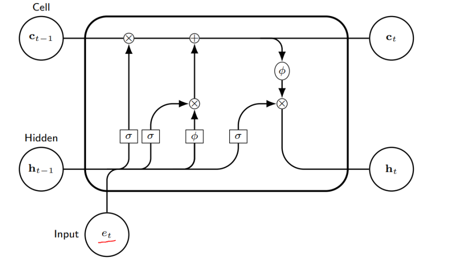


Figure 3: LSTM Cell Structure

Hybrid ARMA-LSTM Model Structure

- Given a sequence of carbon prices $\{y_t\}_{t=1}^{T}$, we first obtain the corresponding predictions $\{\hat{y}_t^A\}_{t=1}^{T}$ from an ARMA model.
- We train an LSTM neural network f^L to model the nonlinear dependency left in the residuals of the ARMA predictions:

$$e_t^A = y_t - \hat{y}_t^A \tag{1}$$

- The predicted residuals $\hat{e}_t^L$ are calculated based on the trained LSTM. The final hybrid model predictions are given by:

$$\hat{y}_t^H = \hat{y}_t^A + \hat{e}_t^L \tag{2}$$

Why This Approach?

- The **ARMA model** retains interpretability while capturing linear dependencies.
- The **LSTM network** improves residual predictions, capturing **non-linear patterns and interactions**.
- This hybrid approach **enhances forecasting accuracy** while maintaining interpretability.

Results

Prediction Approach

- We analyze daily **carbon price predictions** for the **UK and EU ETS**.
- To capture market dynamics, we include **post-2021 data** in training.
- The model is trained on data from **January 2013 to March 2022**, with later data reserved for testing.
- We implement a **rolling prediction procedure**:
  - Model parameters are updated **weekly (every five days)**. The model generates up to 5 steps ahead predictions.
  - Avoids reliance on a **static model**, ensuring adaptation to evolving carbon price trends.

Estimated ARMA Parameters and Optimal ARMA orders

- Despite high correlation between the two price series, their optimal ARMA orders and estimated parameters reveal **distinct time series characteristics**.
- The **optimal ARMA orders** were determined using ‘auto.arima‘ from the R ‘forecast‘ package, minimizing AIC and BIC criteria.
- Initially, both models showed similar structures (AR2, MA2) due to shared training data before the ETS split.
- After the ETS split, **divergence emerged**:
    - The **EU series** follows an **ARMA(2,2)** structure.
    - The **UK series** behaves more like an **MA(2)** model.
- The EU return series maintains a **fixed AR order**, while the UK series has a **fixed MA order**, highlighting structural shifts.

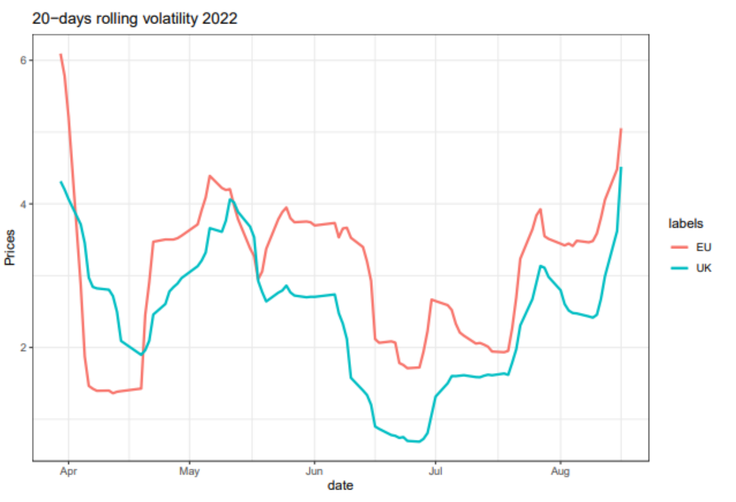
Predictive Performance (MSE): ARMA vs Hybrid Model

- The **hybrid model** performs slightly worse than ARMA on training data but **outperforms it on testing data**.

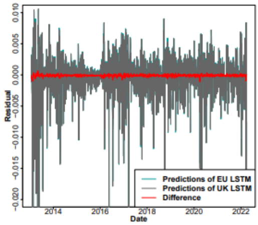
	EU		UK	
	Train	Test	Train	Test
ARMA	0.7887647	5.001231	0.5680096	2.438333
ARMA+LSTM	0.7981942	4.863507	0.5665621	2.365120
AR(1)	0.7810715	4.921645	4.584518	11.589984

Volatility of carbon price on test data for EU ETS and UK ETS

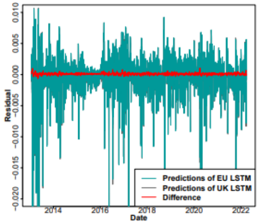
- **Lower MSE in UK data** due to **less volatile** carbon prices compared to the EU market.



How different the two LSTMs are?



(a) Predictions of EU ETS training residuals



(b) Predictions of UK ETS training residuals

Figure 5: Predictions of (a) EU ETS training residuals and (b) UK ETS training residuals from two LSTM network models

How different the two LSTMs are?

- We observe a similar pattern as before: the differences become relatively large when there is a high volatility in the residuals.
- This suggests that the predictions of one market from the LSTM trained on data from the other market cannot capture well the jumps in residuals, compared to those from the LSTM trained on data from the same market.